Syllabus :-

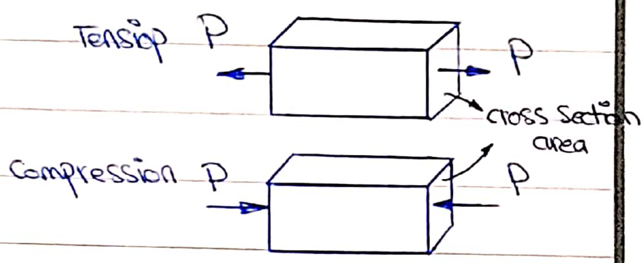
1. Simple Stress and Strain.
2. Compound Bars.
3. Shear force and Bending moment diagrams.
4. Bending
5. Shear Stress distribution.
6. Torsion.
7. Thin cylinder.
8. Thick cylinder.
9. Complex Stresses.

Refrences :-

1. Mechanics of materials. by :- E.J. Hearn Vol. 1
2. Strength of materials. by :- Singer.
3. Mechanic of materials. by :- F. P. Beer.

Strength of Materials

By
Mr. Mazin Y. Abood



Stress $(\sigma) = \frac{P}{A}$

where :-

(σ) :- Normal stress

(i.e) The force is normal to cross-section area

(P) :- Load (N, KN, MN, GN, ...)

(A) :- Cross section area $(m^2, mm^2, \ldots)$

Normal strain :- $(\epsilon) = \frac{\delta L}{L}$

where :- δL :- change in length L :- original length

Also :- $E = \frac{\sigma}{\epsilon} \rightarrow \sigma = E.\epsilon$

where :-

(E) :- Young modulus of elasticity.

$E = \frac{P.L}{A.\delta} \rightarrow \delta = \frac{PL}{AE}$

Poison's Ratio :- (γ)

$\gamma = \frac{\delta d/d}{\delta L/L} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

For most engineering material

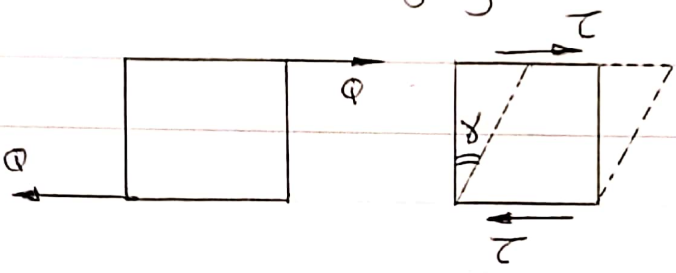
$0,25 < \gamma < 0,33$

Shear stress (τ) :- $\tau = \frac{Q}{A}$

where :- (Q): shear force (A):- Area (Parallel with Q)

Also :- $\tau = G\gamma$

where :- (γ):- Shear strain (G):- Modulus of rigidty



Thermal stresses :-

$\delta = \alpha L \Delta t$

where :- (δ) :- change in length due to the effect of temperature.

(Δt) :- Change in temperature.

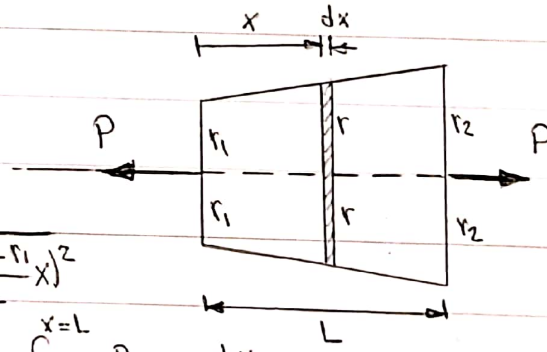
(α) :- Coefficient of linear expansion.

$$\epsilon = \frac{\delta}{L} = \frac{\alpha L \Delta t}{L} = \alpha . \Delta t$$

$$\sigma = E\epsilon = E.\alpha.\Delta t$$

Example (1) :- Drive an expression for the total extension of the bar of circular cross-section as shown when its subjected to an axial tensile load (P).

Solution :-



$$r = r_1 + \frac{r_2 - r_1}{L} x$$

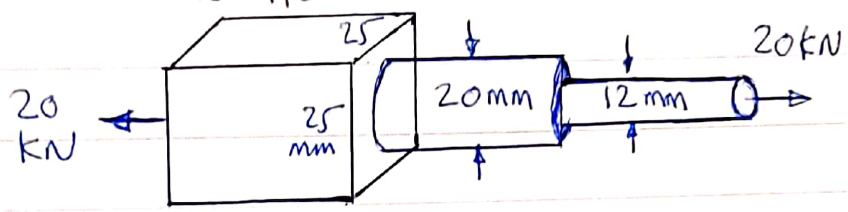
$$\sigma = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{P}{\pi (r_1 + \frac{r_2 - r_1}{L} x)^2}$$

For (dx) :- $d\delta = \frac{P.dx}{AE} \Rightarrow \delta = \int_{x=0}^{x=L} \frac{P}{\pi E} * \frac{dx}{(r_1 + \frac{r_2 - r_1}{L} x)^2}$

$$\Rightarrow \delta = \frac{P.L}{\pi E r_1 r_2}$$

P.T.O.

Example (2) :- Find the stress in each section and total extension ? $E = 210\ GPa$



Solution :-

① Section (AB) :- $\sigma = \frac{P}{A} = \frac{20 \times 10^{3}}{25 \times 25 \times 10^{-6}} = 32\ \frac{MN}{m^2} = 32\ MPa$

② Section (BC) :- $\sigma = \frac{P}{A} = \frac{20 \times 10^{3}}{\frac{\pi}{4}(20 \times 10^{-3})^2} = 63.662\ MPa$

③ Section (CD) :- $\sigma = \frac{P}{A} = \frac{20 \times 10^{3}}{\frac{\pi}{4}(12 \times 10^{-3})^2} = 176.838\ MPa.$

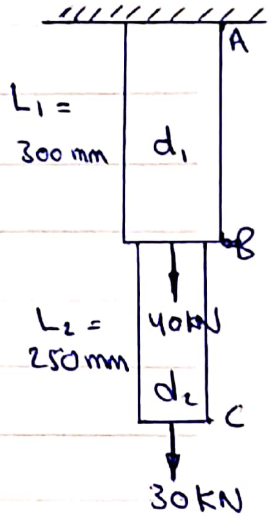
$\delta_T = \delta_{AB} + \delta_{BC} + \delta_{CD}$

$$= \frac{P.L_{AB}}{A_{AB} \cdot E} + \frac{P.L_{BC}}{A_{BC} \cdot E} + \frac{P.L_{CD}}{A_{CD} \cdot E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} + \frac{L_{CD}}{A_{CD}}\right)$$

$$= \frac{20 \times 10^{3}}{210 \times 10^{9}}\left(\frac{0.05}{25 \times 25 \times 10^{-6}} + \frac{0.04}{\frac{\pi}{4}(20 \times 10^{-3})^2} + \frac{0.05}{\frac{\pi}{4}(12 \times 10^{-3})^2}\right)$$

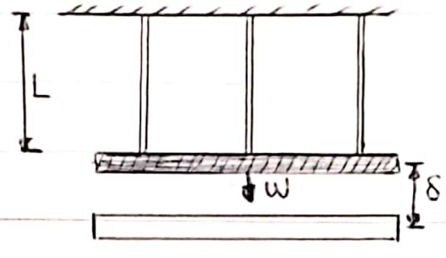
$= 0.0618\ mm.$

Example 8- Two Solid cylinderical rods AB and BC are welded together and loaded as shown. Knowing that the avarage normal stress not exceed 175 Mpa in rod AB and 150 Mpa in rod BC. Determine the Smallest value of d_1 and d_2



Sol 8- in lecture.

External Load :-



From free body diagram :-

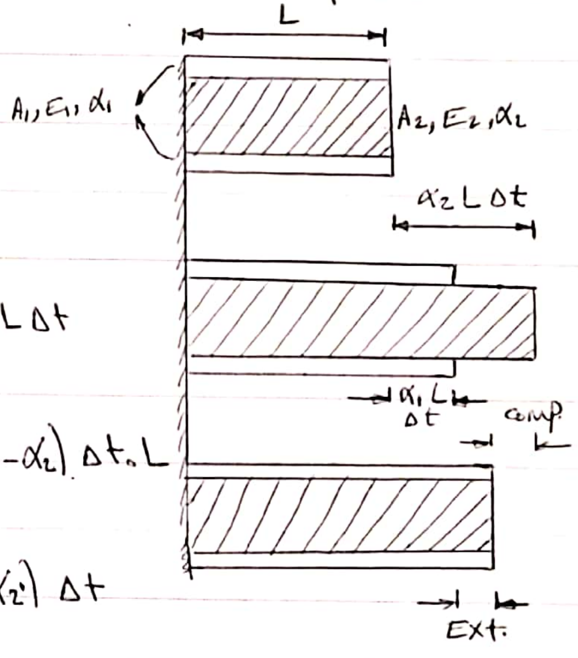
$2F_1 + F_2 = W$

$\delta_1 = \delta_2 \Rightarrow \dfrac{F_1 \cdot L_1}{A_1 \cdot E_1} = \dfrac{F_2 \cdot L_2}{A_2 \cdot E_2}$

$L_1 = L_2$

$\Rightarrow \dfrac{F_1}{A_1 \cdot E_1} = \dfrac{F_2}{A_2\, E_2}$

A_1, E_1, A_2 and E_2 are known, thus two equation with two unknown (F_1, F_2)



Temperature change :-

$2F_1 = F_2$

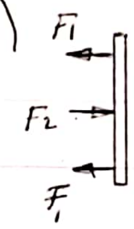
Comp. + Ext. $= \alpha_1 L \Delta t - \alpha_2 L \Delta t$

$\delta = \dfrac{FL}{AE}$.

$\Rightarrow \dfrac{F_1 L}{A_1 E_1} + \dfrac{F_2 L}{A_2 E_2} = (\alpha_1 - \alpha_2)\, \Delta t \cdot L$

$\Rightarrow \dfrac{F_1}{A_1 E_1} + \dfrac{F_2}{A_2 E_2} = (\alpha_1 - \alpha_2)\, \Delta t$

Two equation with two unknowns (F_1 and F_2)



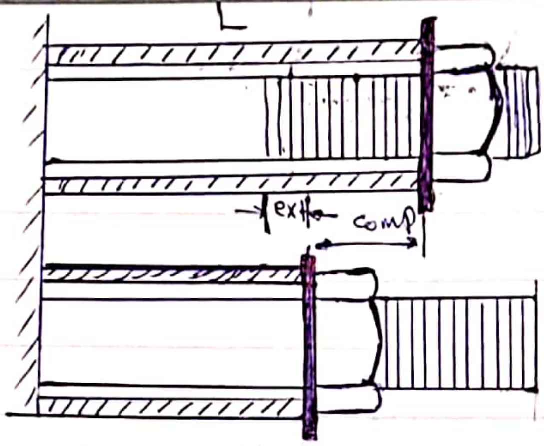
Tube and rod :-

From free body diagram :-

$$F_t = F_r$$

$$\text{Comp.} + \text{Ext.} = d$$

$$\therefore \frac{F_t \cdot L}{A_t E_t} + \frac{F_r \cdot L}{A_r \cdot E_r} = d$$



Two equation with two unknown (F_t and F_R)

Example (1) :- A compound bar consist of four wires of brass, the diameter of each one (2.5 mm) and one wire of steel has a diameter of (1.5 mm) determine :-

① The stress in each wires if the bar is subjected to load of (500N). All wires have a same length

② The common deflection if ($L = 0.75$ m). $E_s = 200$ GPa $E_B = 100$ GPa

Solution :- $\sum F_y = 0 \rightarrow 4F_B + F_s = 500 \rightarrow ①$

$$\delta_B = \delta_s \Rightarrow \frac{F_B \cdot L}{A_B \cdot E_B} = \frac{F_s \cdot L}{A_s \cdot E_s}$$

$$\therefore \frac{F_B}{\frac{\pi}{4}(2.5 \times 10^{-3})^2 \times 100 \times 10^9} = \frac{F_s}{\frac{\pi}{4}(1.5 \times 10^{-3})^2 \times 200 \times 10^9}$$

$$\Rightarrow \frac{F_B}{(2.5)^2} = \frac{F_s}{(1.5)^2}$$

. Two equation with two unknowns F_b and F_s

$$\Rightarrow F_s = 76 \text{ N} \qquad F_B = 106 \text{ N}$$

$$\sigma_B = \frac{F_B}{A_B} = \frac{106}{\frac{\pi}{4}(2.5)^2 \times 10^{-6}} = 21.6 \text{ MPa} \qquad \sigma_s = \frac{F_s}{A_s} = \frac{76}{\frac{\pi}{4}}(1.5 \times 10^{-3})^2 = 43.2 \text{ MPa}$$

$$\delta_s = \delta_B = \frac{F_s \cdot L}{A_s E_s} = \frac{43.2 \times 10^6 \times 0.75}{200 \times 10^9} = 0.162 \text{ mm}$$

Example (2) :- A compound bar is constructed from three bars 50mm wide by 12mm thick together to form a bar 50mm wide 36mm thick. The middle bar is of aluminum alloy for which E = 70 GPa and the outside of brass with E = 100 MPa. If the bars are initially fastened at 18°C and the temperature of the whole assembly is then raised to 50°C, determine the stresses set up in the brass and the aluminum.

$\alpha_B = 18 * 10^{-6} \frac{1}{C^\circ}$ $\alpha_A = 22 * 10^{-6} \frac{1}{C^\circ}$

Solution :- $2F_B = F_{AL}$

Ext. + Comp. $= (\alpha_{Al} - \alpha_B) L \cdot \Delta t$

$$\frac{F_B \cdot L}{A_B\, E_B} + \frac{F_{AL} \cdot L}{A_{AL} \cdot E_{AL}} = (\alpha_{AL} - \alpha_B) L \Delta t$$

$$\frac{F_B}{50*12*10^{-6} * 100*10^9} + \frac{F_{AL}}{50*12*10^{-6} * 70*10^9} = (22-18)*10^{-6}(50-18)$$

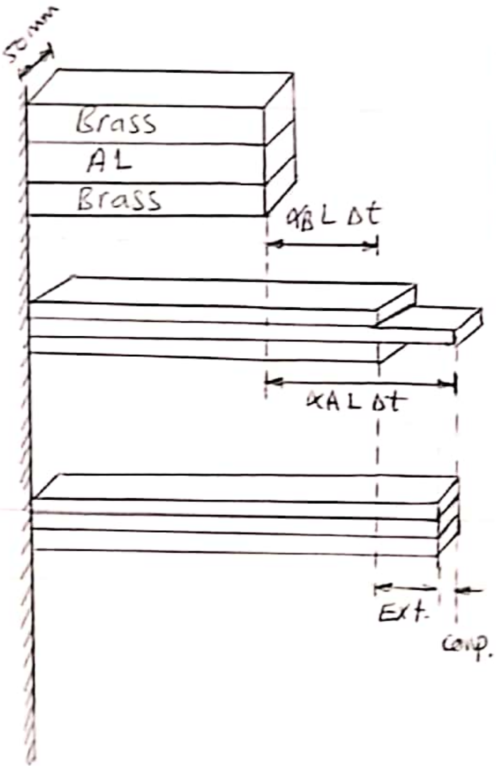
From these equation :-

$F_{AL} = 3984$ N

$F_B = 1992$ N

$$\sigma_B = \frac{F_B}{A_B} = \frac{1992}{50*12*10^{-6}} = 3.32 \text{ MPa (Ten)}$$

$$\sigma_{AL} = \frac{F_{AL}}{A_{AL}} = \frac{3984}{50*12*10^{-6}} = 6.64 \text{ MPa (comp)}$$



* If P = 15 KN (comp) Find σ_B and σ_{AL}

$F_A + 2F_B = 15$

$$\delta_B = \delta_{AL} \Rightarrow \frac{F_{AL} \cdot L}{A_A\, E_A} = \frac{F_B \cdot L}{A_B \cdot E_B}$$

$\Rightarrow F_B = 5.56\ KN \qquad F_{AL} = 3.89\ KN$

$\sigma_{AL} = \dfrac{F_{AL}}{A_{AL}} = 6.48\ MPa \qquad \sigma_B = 9.26\ MPa$

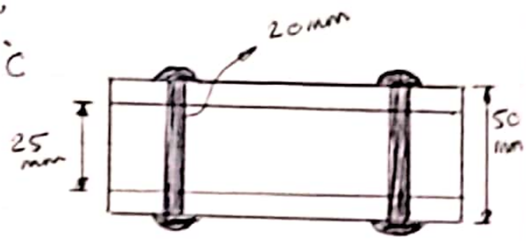
$\Rightarrow \sigma_B\ \text{Total} = -9.26 + 3.32\ MPa \quad (\text{Comp})$

$\sigma_{AL}\ \text{total} = -6.48 - 6.64 = 13\ MPa\ \ \text{Comp})$

Example (3) :- A compound bar is constructed from a steel rod of 25mm diameter surrounded by a copper tube of 50 mm outside diameter and 25 mm inside diameter. The rod and tube are joined by two diameter pins as shown in the figure. Find the stress set up in the pins if, after pinning the temperature is raised by 50 °C.

For steel: $E = 210\ GPa \quad \alpha = 11 \times 10^{-6}\ 1/°C$

For copper :- $E = 105\ GPa \quad \alpha = 17 \times 10^{-6}\ 1/°C$



Solution:- $F_T = F_R \rightarrow (1)$

$\delta_T + \delta_R = (\alpha_T - \alpha_R)\, L\, \Delta t$

$\Rightarrow \dfrac{F_T . L}{A_T . E_T} + \dfrac{F_R . L}{A_R . E_R} = (\alpha_T - \alpha_R)\, L . \Delta t$

$\Rightarrow \dfrac{F_T}{\frac{\pi}{4}(50^2 - 25^2) \times 10^{-6} \times 105 \times 10^9} + \dfrac{F_R}{\frac{\pi}{4}(25)^2 \times 10^{-6} \times 210 \times 10^9} = (17-11) \times 10^{-6} \times 50$

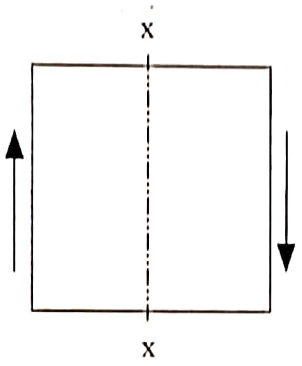
$\Rightarrow 6.46\, F_T \times 10^{-9} + 9.7\, F_R \times 10^{-9} = 300 \times 10^{-6} \rightarrow (2)$

From these equations :-

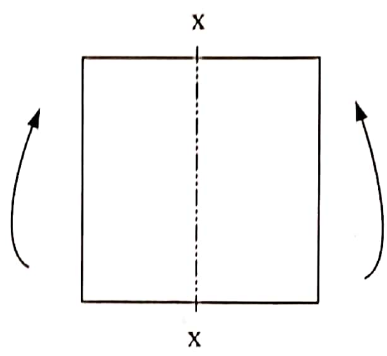
$F_T = F_R = 18.564\ KN$

$\tau = \dfrac{Q}{2A} = \dfrac{18.564}{\frac{\pi}{4}(20)^2 \times 10^{-6} \times 2} = 29.55\ MPa.$

Shear force and bending moment diagram

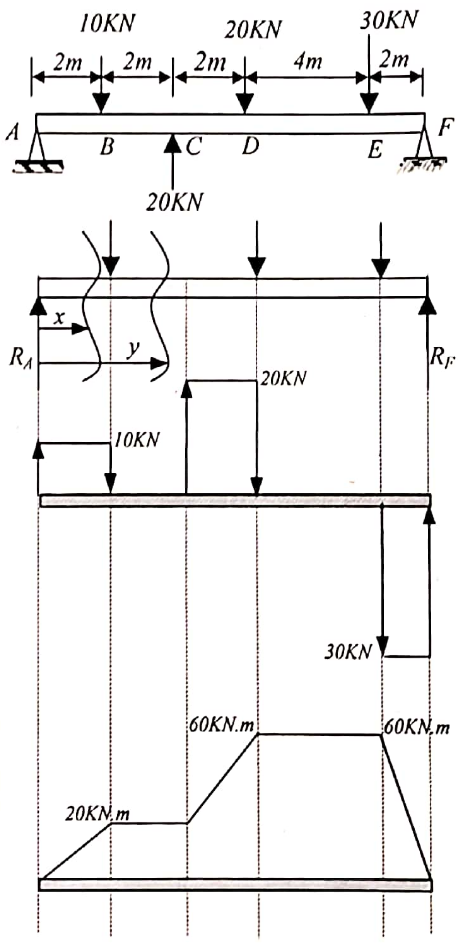


Positive S.F



Positive B.M

Concentrated loads:



$R_A(12) = 10(10) + 20(6) + 30(2) - 20(8) \Rightarrow R_A = 10KN$

$\sum F_Y = 0 \Rightarrow R_A + R_F = 10 + 20 + 30 - 20$

$\Rightarrow R_F = 30KN$

section x;

$M_X = R_A \cdot X = 10x$

(increasing line)

section y;

$M_Y = R_A \cdot Y - 10(Y - 2)$

$10 \cdot Y - 10Y + 20 = 20$

(horizontal line)

- *Notes*:
1. *Values of shear force =Slope of bending moment.*
2. *Area of shear force diagram between two points =value difference between bending moment of these points.*

Uniformly distributed loads:

$$\sum M_B = 0$$

$$R_A * 12 = 25 * 12 * 6 \Rightarrow R_A = 150KN$$

$$\sum FY = 0$$

$$R_A + R_B = 25 * 12 \Rightarrow R_B = 150KN$$

Section x:

S.FX=150-25X(decreasing line)

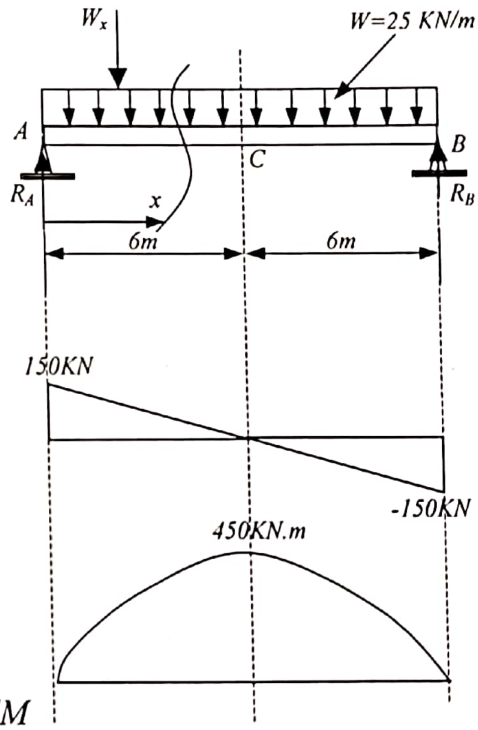
$$B*M_x = RA * x - W * x * \frac{x}{2} = 150X - 125x^2$$

Convex parabola

To find $B*M_{max}$

$$\frac{dM}{AX} = 0 \Rightarrow 150 - 125X = 0 \Rightarrow X = 6m$$

$$\Rightarrow B*M_{max} = 450KN.m$$

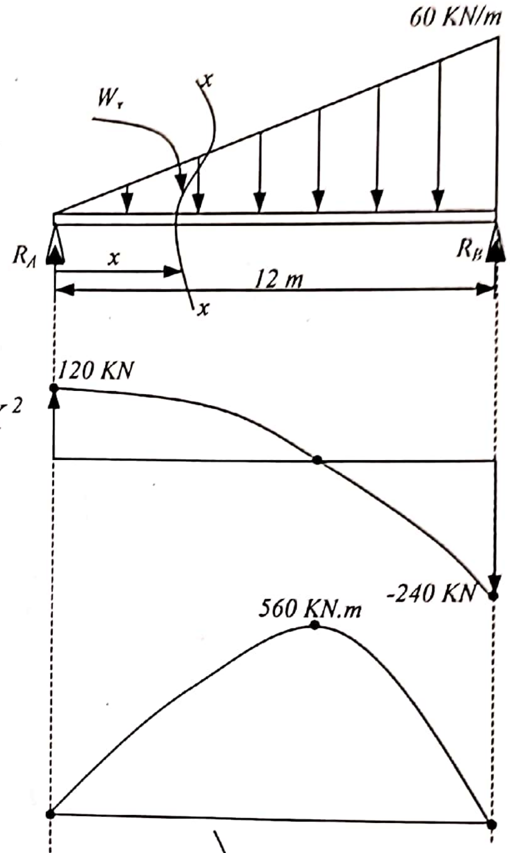


$$Note: S*F_x = Q_x = 150 - 125X = \frac{dM}{dx} \Rightarrow Q = \frac{dM}{dx}$$

M= Q dx

$$IFQ = 0 \Rightarrow M \text{ is minimum or maximum}$$

∢ Distributed loads of increasing value



$$\sum MB = 0 \Rightarrow RA * 12 = \frac{60*12}{2}(300)*4 \therefore RA = 120KN$$

$$\sum FY = 0 \Rightarrow RA + RB = 60 * \frac{12}{2} \Rightarrow RB = 240KN$$

SECTION X:

$$Q_X = RA - \frac{1}{2}W_X(X) = 120 - \frac{1}{2}\left(\frac{60}{12}*X\right)X = 120 - 2.5X^2$$

(Convex parabola)

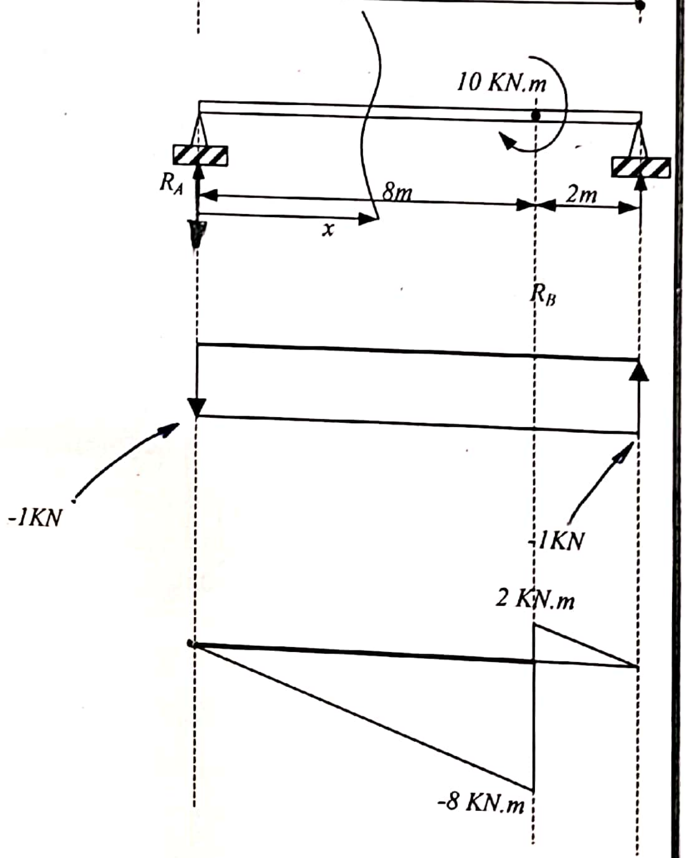
$$Qx = 0 \Rightarrow X = \sqrt{48}$$

$$Mx = RA * X - \left(\frac{1}{2}(5X)X\right)\frac{X}{3} = 120X - \frac{5}{6}X^3$$

(Convex third degree)

$$M_{max} at Qx = 0 at x = \sqrt{48}$$

$$M_{max} = 120(\sqrt{48}) - \frac{5}{6}(\sqrt{48})^3 = 560KN.m$$



∢ Couples:

$$\sum MB = 0 \Rightarrow RA * 10 = 10 \Rightarrow RA = 1KN$$

$$\sum FY = 0 \Rightarrow RA = RB \Rightarrow RB = 1KN$$

$Qx = -RA = -1$

$Mx = -RA.X = -X$

$Mc = -8$

Note: point of inflexion occur at (M=0)

Example-1-

$$\sum ME = 0 \Rightarrow RA * 8 + 40 * 2$$
$$= 10 * 2 * 7 + 20 * 6 + 20 * 3 + 10(1) + 20(3)(1.5)$$
$$\Rightarrow RA = 42.5KN$$
$$\sum FY = 0 \Rightarrow RA = RE = 10(2) + 20 + 20 + 10 + 20(3) + 40 \Rightarrow RE = 127.5KN$$

$$QB = 42.5 - 10(2) - 20 = 2.5\ KN$$
$$QD = 22.5 - 20 - 20 - 20(2) - 10 = 67.5KN$$

$$MX = 42.5X - 10X\left(\frac{X}{2}\right)$$

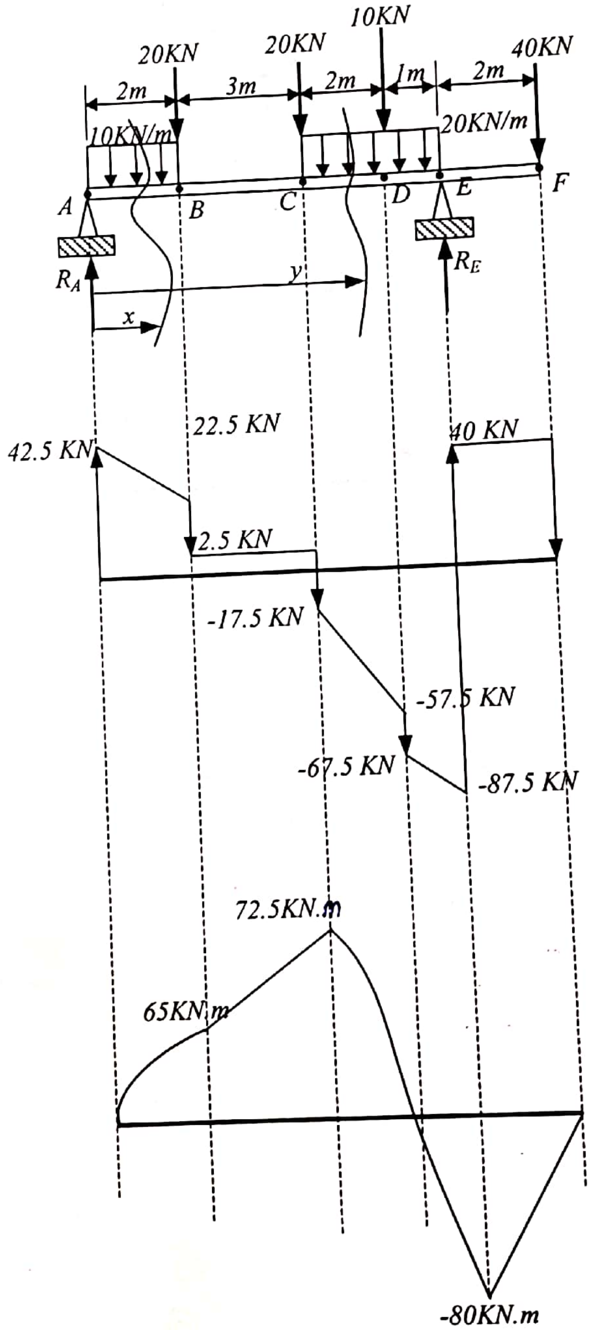
$$MB = 42.5(2) - 10\left(\frac{22}{2}\right) = 26\,KNm$$

$$MY = RA(Y) - 10 * 2(Y-1) - 20(Y-2) -$$
$$20(Y-5) - 20(Y-5)\left(\frac{Y-5}{2}\right) = -90 + 82.5Y - 10Y^2$$

$$MY = 0 \Rightarrow 10Y^2 - 82.5Y + 90 = 0$$
$$\Rightarrow Y = 6.96\ or y = 1.5(neglect)$$

3

$$MC = -90 + 82.5(5) - 10(5)^2 = 72.5\ KNM$$
$$ME = -90 + 82.5(8) - 10(8)^2 - 10(1) = -80\ KN.M$$



Example 2

C=80KN.M

$\sum MB = 0 \Rightarrow Rc(6) + 10*3*1.5 + 80 = \frac{1}{2}*6*48*2 \Rightarrow Rc = 27\ KN$

$\sum FY = 0 \Rightarrow Rc + RB = 10*3 + \frac{1}{2}*48*6 \Rightarrow RB = 147\ KN$

$QX = -Rc + \frac{1}{2}wx.x = 27.2 + \frac{1}{2}(48)\frac{x}{6}.x = -27.2 + 4x^2$

(Concave parabola)

$Qx = 0 \Rightarrow 4X^2 - 27.2 = 0 \Rightarrow x = 2.61\,m$

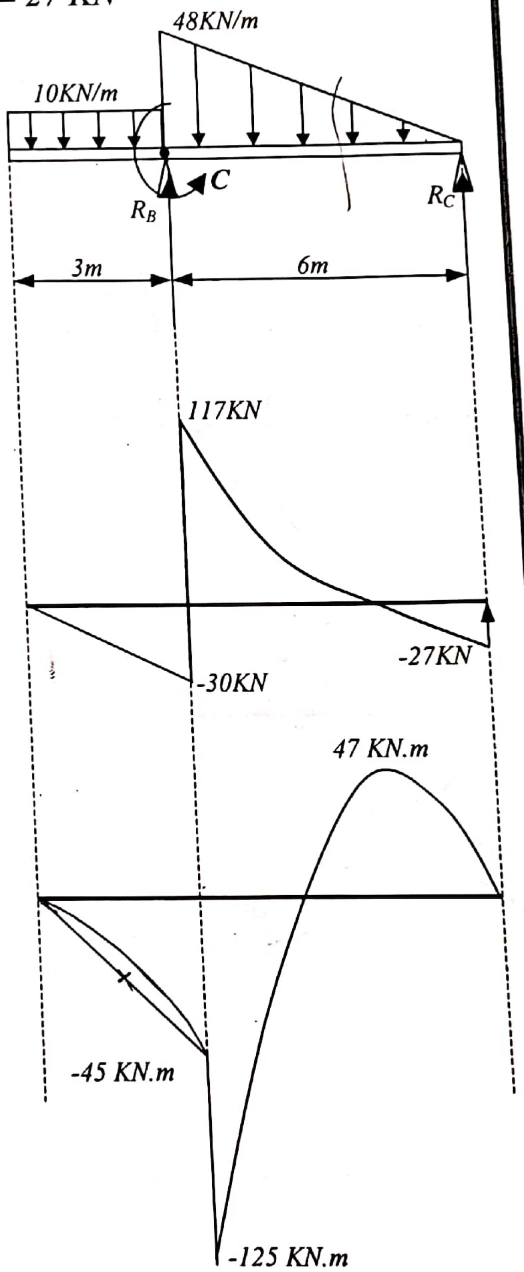
$Mx = 27.2x - \frac{1}{2}w.x.\frac{x}{3} = 27.2x - \frac{1}{2}(8x)*x*\frac{x}{3}27.2x - \frac{3}{4}x^3$

(Convex third degree)

$M_{max} = 27.2x(2.61) - \frac{3}{4}(2.61)^3 = 47.3\ KN.m$

$M = 0 \Rightarrow \frac{3}{4}x^3 - 27.2x = 0 \Rightarrow x = 4.5\,m$

$Mc = 27.2(6) - \frac{3}{4}(6)^3 = -125\ KN.m$



Example: A simply supported beam (ABC) has length of (6m) and loaded as shown below. The weight of the beam 100N/m. find:
(1) The reactions at A and B
(2) Draw shear force diagrams and from it find the value and position of maximum bending moment
Solution:

$$\sum MB = 0 \Rightarrow RA*5 - 100*5*2.5 + 100*1*0.5 = 0$$
$$\Rightarrow RA = 240\,N$$

$$\sum FY = 0 \Rightarrow RA + RB = 100*6 \Rightarrow RB = 360N$$

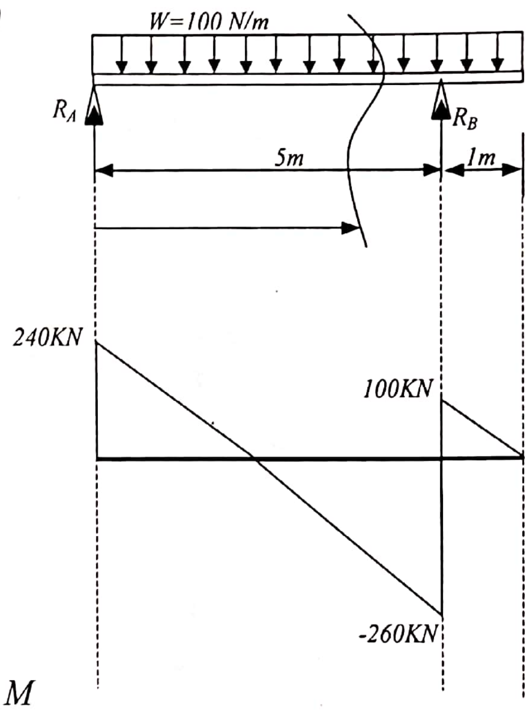
$$Mx = RA*x - 100x*\frac{x}{2} = 240x - 50x^2$$

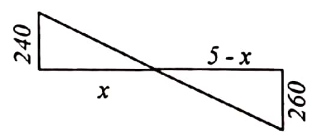
$$Rx = RA - 100x \Rightarrow Rx = 240 - 100x \Rightarrow$$
$$M_{max}\,atR = 0 \Rightarrow 240 - 100x = 0 \Rightarrow x = 2.4\,m$$
$$M_{max} = 240(2.4)*50(2.4)^2 = 288N.M$$

From (S.F.D)

$$\frac{240}{x} = \frac{260}{5-x} \Rightarrow x = 2.4\,m$$

$$B.M_{max} = areaof(S.F.D) = \frac{1}{2}*240*2.4 = 288\,N.M$$



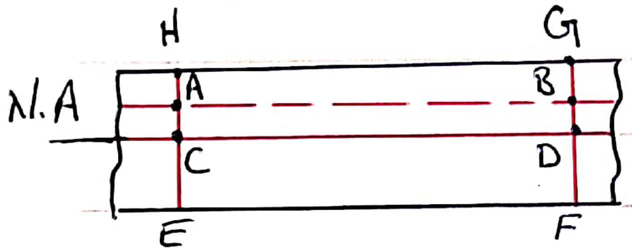


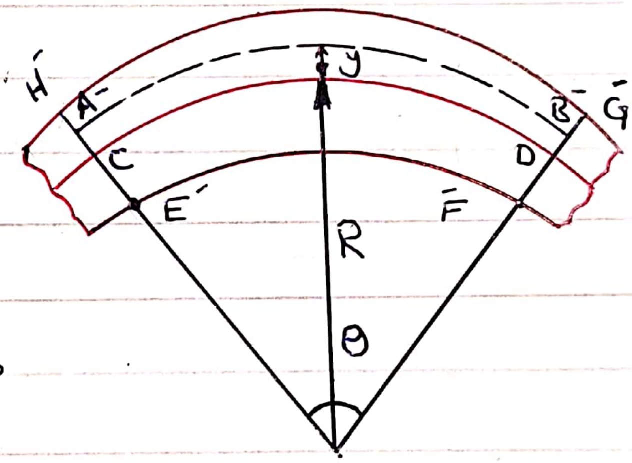
Chapter Four :- Bending.

* Simple bending theory :-

* Assumptions :-

1. The beam is initially straight and unstressed.
2. The material is homogenous and isotropic.
3. The Elastic limit is nowhere exceeded.
4. Young's modulus is the same in tension and compression.
5. Plane cross-section remains plane after bending.
6. Every cross-section is symmetrical about the plane of bending.
7. the loading is pure bending.





The strain of the line AB

$$\epsilon = \frac{A'B' - AB}{AB}$$

before bending $AB = CD$

and CD on the nutral axis (No strain)

$$\therefore e = \frac{A'B' - CD}{CD} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

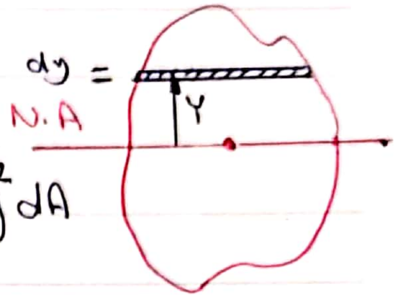
$$\text{But } \epsilon = \boxed{\frac{\sigma}{E} = \frac{y}{R}} \rightarrow (1)$$

Also 8- $F = \sigma \cdot A$

$\Rightarrow dF = \sigma \cdot dA \Rightarrow dM = Y \cdot dF$

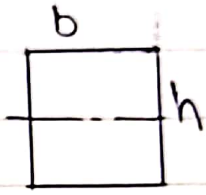
$= \sigma Y \cdot dA$

$\therefore M = \int_A \sigma Y dA = \int_A \frac{E}{R} Y^2 dA = \frac{E}{R} \int_A y^2 dA$



$\therefore M = \frac{E}{R} \cdot I \rightarrow$ (2) $\quad I \equiv$ Second moment of inertia

I (rectangular) $= \frac{bh^3}{12}$



I (circle) $= \frac{\pi}{4} R^4$ where (R) = radius of circle

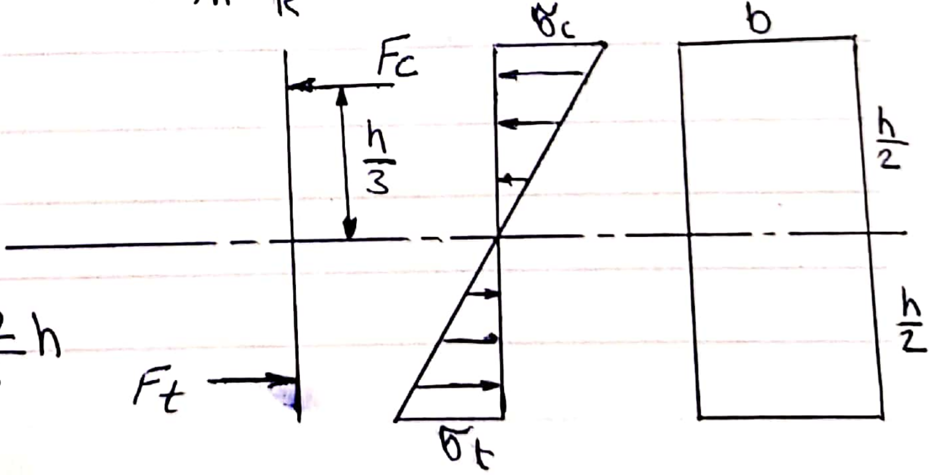
* Always the nutral axis (N.A) passes through the Centroid because assumption (7) means that there is no resultant force across the Section, i.e

$F = \int_A \sigma dA = 0 \Rightarrow \int_A \frac{E}{R} Y dA = 0 \Rightarrow \int_A dA = 0$

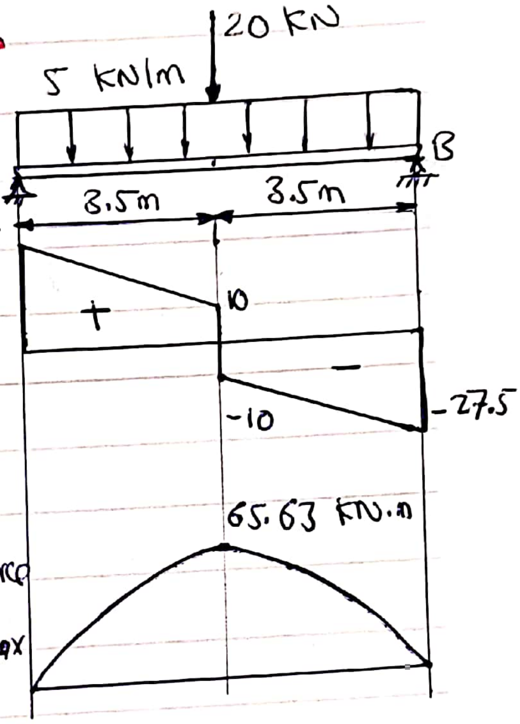
$F_c = F_t$

$M = F_t \cdot \frac{2}{3} h$

$= (\frac{1}{2} \sigma_t \cdot \frac{h}{2} b) \frac{2}{3} h$



Example (1) :- the beam loaded as shown, Find the maximum bending stress (σ_{max}).



Solution :-

$\Sigma M_B = 0$

$R_A * 7 = 5*7*3.5 + 20*3.5$

$\Rightarrow R_A = 27.5$ KN

$\Sigma F_y = 0 \Rightarrow R_A + R_B = 5*7+20$

$\Rightarrow R_B = 27.5$ KN

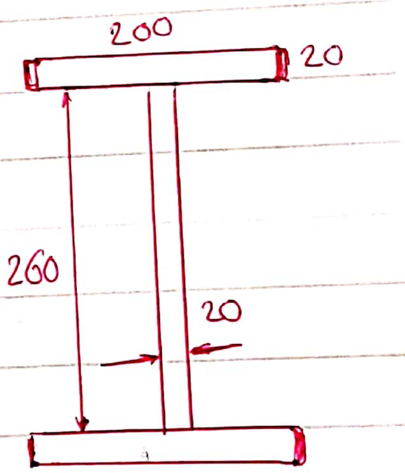
M_{max} occures at zero shear force

There are two ways to find M_{max}

① $M_{max} = 27.5*3.5 - 5*3.5*\frac{3.5}{2} = 65.62$ KN.m

② M_{max} = area of shear force between A and the Centre

$= \frac{1}{2}*17.5*3.5 + 10*3.5 = 65.625$ KN.m



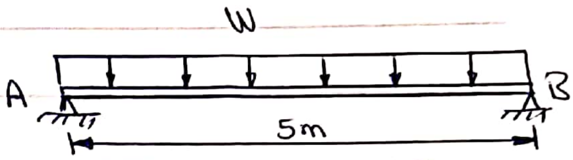
$Y_{max} = 150$ mm

$$I_{N.A} = \frac{200\,(300)^3}{12} - 2*\frac{90*260^3}{12}$$

$= 1.86*10^8$ mm^4

$$\sigma = \frac{65.625*10^3*150*10^{-3}}{1.86*10^{-4}} = 51.8 \text{ Mpa.}$$

Example (2):– Find (w) if maximum stress in tension is 160 MPa and in compression is 80 MPa.



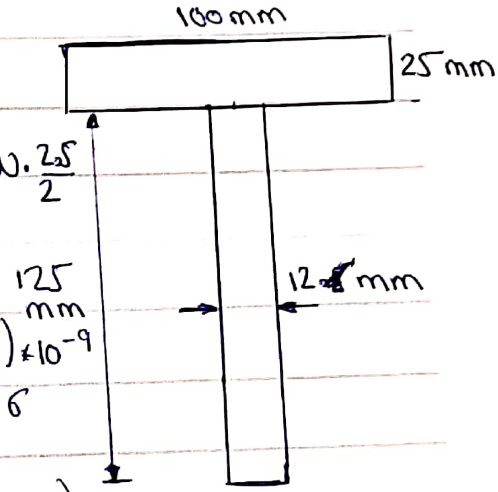
Solution:–

Because of the Symmetry

$R_A = R_B = \frac{1}{2} wL$

Maximum bending moment occurs at the Centre $= 2.5W * 2.5 - 2.5W \cdot \frac{2.5}{2}$

$= 3.125 W$



$$\bar{y} = \frac{\Sigma AY}{\Sigma A} = \frac{(100*25*137.5 + 125*12*62.5)*10^{-9}}{(100*25 + 125*12)*10^{-6}}$$

$= 109.4$ mm (From the base)

$$I_{N.A} = \left[\frac{100*40.6^3}{3} - \frac{88*15.6^3}{3} + \frac{12*109.4^3}{3}\right]*10^{-12} = 7.36*10^{-6}\ m^4$$

$$M = \frac{\sigma \cdot I}{Y} = \frac{80*10^6 * 7.36*10^{-6}}{40.6*10^{-3}} = 14.5 \text{ KN.m}$$

$$M = \frac{160*10^6 * 7.36*10^{-6}}{109.4*10^{-3}} = 10.76 \text{ KN.m}$$

$\therefore M_{max} = 10.76$ KN.m $= 3.125 W$

$\Rightarrow W = 3.44$ KN/m

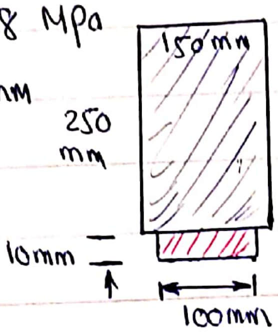
Example ② :- Determine the Concentrated load that can be applied at the Centre of a simply supported span 6 m long if $\frac{E_s}{E_w} = 20$. $(\sigma_{max})_s = 120$ Mpa

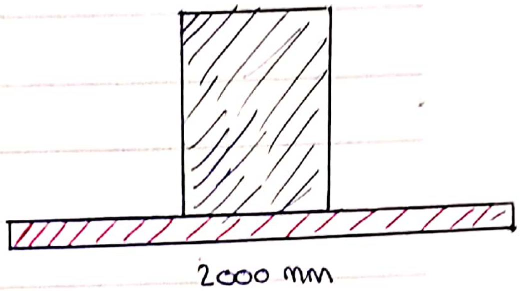
$(\sigma_{max})_w = 8$ Mpa

$\bar{y} = 170.2$ mm

From top

$I = 416 \times 10^{-6}$ m^4





Solution :-

$$E_s t_s = E_w t_w \Rightarrow \frac{E_s}{E_w} t_s = 20 \times 100 = 2000 \text{ mm}$$

$$\sigma_w = \frac{M.Y}{I} \Rightarrow M = \frac{\sigma . I}{Y} \Rightarrow M = \frac{8 \times 10^{6} \times 416 \times 10^{-6}}{170.2 \times 10^{-3}} = 19.55 \text{ kN.m}$$

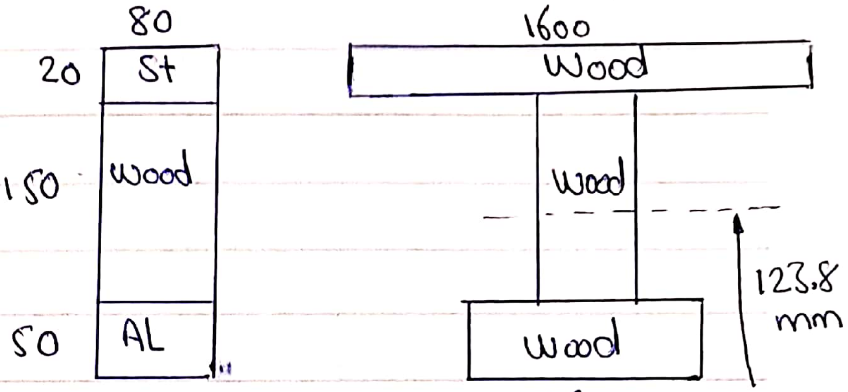
$$\sigma_w = \sigma_s . \frac{E_w}{E_s} = 120 \times \frac{1}{20} = 6 \text{ Mpa.}$$

$$M = \frac{6 \times 10^{6} \times 416 \times 10^{-6}}{89.78 \times 10^{-3}} = 27.8 \text{ kN.m}$$

$$M_{max} = 19{,}55 = \frac{W}{2} \times 3 \Rightarrow W = 13.03 \text{ kN}$$

Example (3) :- Find the maximum bending moment for a Composite beam that has the following properties.
$(\sigma_{max})_s = 120$ Mpa , $(\sigma_{max})_{Al} = 80$ Mpa . $(\sigma_{max})_w = 10$ Mpa
$E_s = 200$ Gpa , $E_{AL} = 70$ Gpa and $E_w = 10$ Gpa.

Solution:-



$E_s t_s = E_w t_w \Rightarrow t_w = \frac{200}{10} * 80 = 1600$ mm
$E_w t_w = E_{AL} \cdot t_{AL} \Rightarrow t_w = \frac{70}{10} * 80 = 560$ mm

$$\bar{y} = \frac{\Sigma AY}{\Sigma A} = \frac{20*1600*210 + 150*80*125 + 560*50*25}{20*1600 + 150*80 + 560*50} = 123.8 \text{ mm}$$

$$I = \frac{560*123.8^3}{3} - \frac{480*73.8^3}{3} + \frac{1600*96.2^3}{3} - \frac{1520*76.2^3}{3} = 540.5 * 10^{-6} \text{ m}^4$$

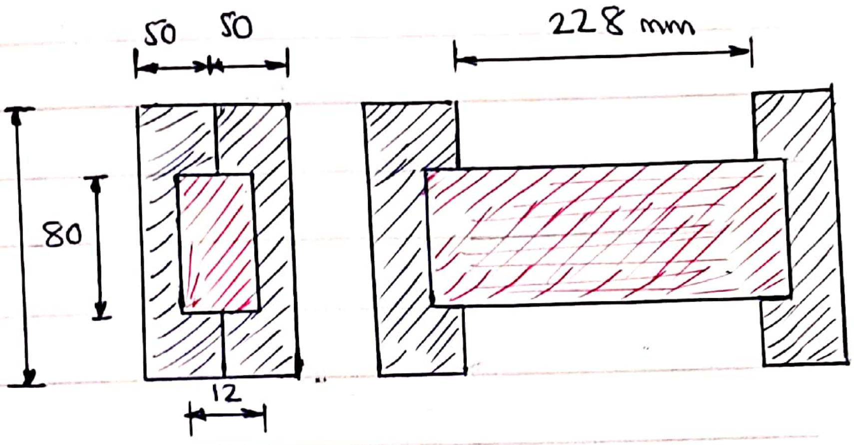
$$M = \frac{10*10^6 * 540.5*10^{-6}}{76.2*10^{-3}} = 70.8 \text{ KN.m}$$

$$\sigma_w = \frac{E_w}{E_s} \cdot \sigma_s = \frac{10}{200} * 120 = 6 \text{ MPa} \Rightarrow M = \frac{6*10^6 * 540*10^{-6}}{96.2*10^{-3}} = 33.7 \text{ KN.m}$$

$$\sigma_w = \frac{E_w}{E_{al}} \cdot \sigma_{al} = \frac{10}{70} * 8 = 11.4 \text{ MPa} \Rightarrow M = \frac{11.4*10^6 * 540*10^{-6}}{123.8*10^{-3}} = 49.5 \text{ KN.m}$$

$\therefore M_{max} = 33.7$ KN.m

Example ① :-



$M_{max} = ?$

For wood :-

$\sigma_{max} = 12$ MPa

$E = 10$ GPa

For steel :-

$\sigma_{max} = 150$ MPa $E = 200$ GPa.

Solution :-

$$E_s t_s = E_w t_w \Rightarrow t_w = \frac{E_s}{E_w} \cdot t_s = \frac{200}{10} * 12 = 240 \text{ mm}$$

$$I_{N.A} = 2 * \frac{50 * 200^3}{12} + \frac{228 * 80^3}{12} = 76.36 * 10^6 \text{ mm}^4 = 76.36 * 10^{-6} \text{ m}^4$$

$$M_{max} = \frac{\sigma_{max} \cdot I}{Y_{max}} = \frac{12 * 10^6 * 76.36 * 10^{-6}}{100 * 10^{-3}} = 9200 \text{ N.m}$$

$$\frac{\sigma_s}{\sigma_w} = \frac{E_s}{E_w} \Rightarrow \sigma_w = \frac{E_w}{E_s} \cdot \sigma_s = \frac{10 * 10^9}{200 * 10^9} * 150 * 10^6$$

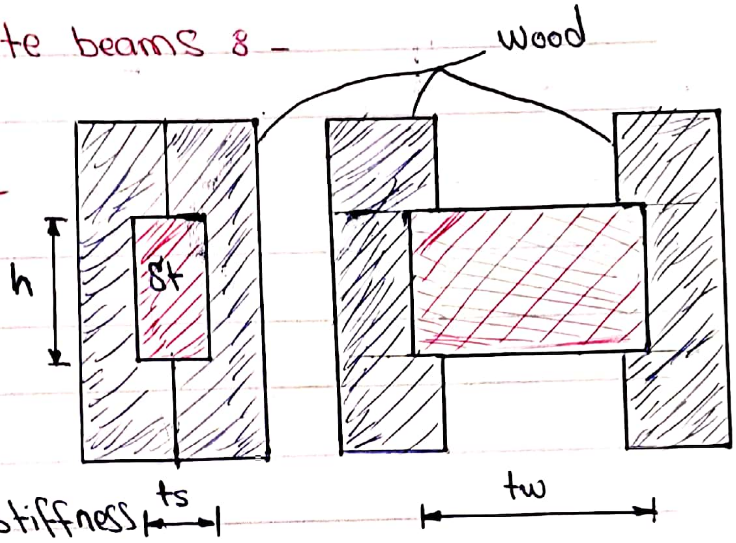
$$\Rightarrow 7.5 \text{ MPa}.$$

$$M_{max} = \frac{7.5 * 10^6 * 76.36 * 10^{-6}}{40 * 10^{-3}} = 14.3175 \text{ KN.m}$$

$\therefore M_{max} = 9200$ N.m

* Bending of Composite beams :-

(1) Flitched beams:-



$M = EI \cdot \frac{1}{R}$

$\therefore M \propto \frac{1}{R}$

where EI = constand

and called Flexural stiffness

Equivalent section (EI) constant

i.e $E_s I_s = E_w I_w \Rightarrow E_s \cdot \frac{t_s \cdot h^3}{12} = E_w \cdot \frac{t_w h^3}{12}$

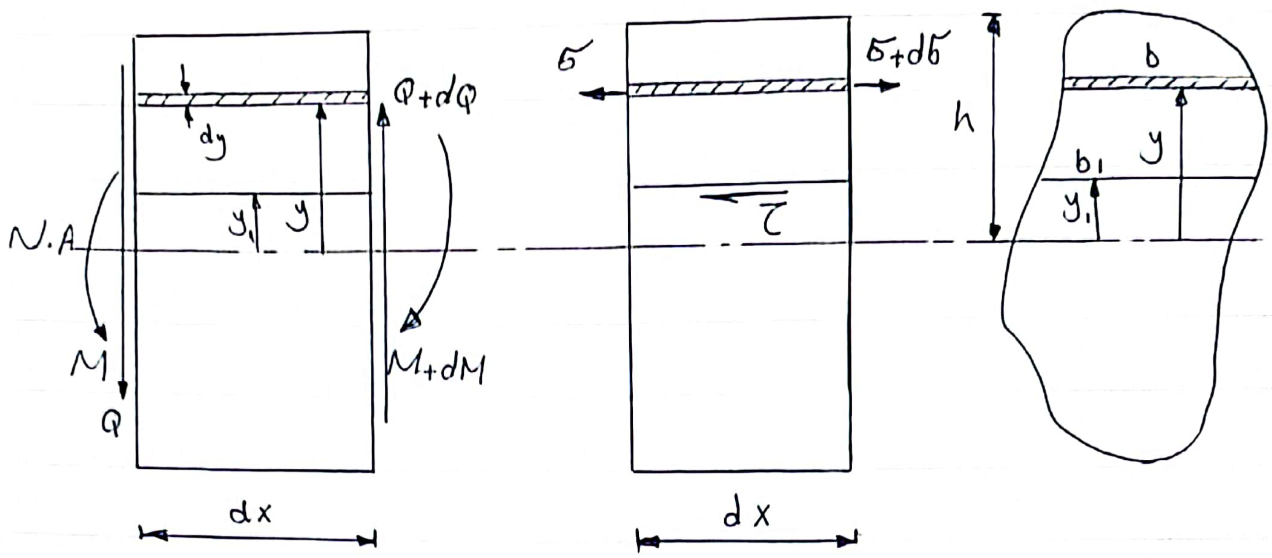
h is constant thus

$$E_s \cdot t_s = E_w \cdot t_w \rightarrow (1)$$

and $R_w = R_s \Rightarrow \frac{E_s \cdot Y_s}{\sigma_s} = \frac{E_w Y_w}{\sigma_w}$

$$y_s = y_w \Rightarrow \frac{E_s}{\sigma_s} = \frac{E_w}{\sigma_w} \rightarrow (2)$$

Equations (1) and (2) are very important to solve any proplem for this type of beams.



$$\sigma = \frac{M \cdot y}{I}, \qquad \sigma + d\sigma = \frac{(M+dM)\,y}{I} \Rightarrow d\sigma = \frac{dM \cdot y}{I}$$

$$\tau \cdot b_1 \cdot dx = \int_{y_1}^{h} d\sigma \cdot b \cdot dy = \int_{y_1}^{h} \frac{dM}{I}\, b \cdot y \cdot dy$$

But $dM = Q\,dx \Rightarrow \tau \cdot b_1 \cdot dx = \int_{y_1}^{h} \frac{Q \cdot dx}{I}\, b \cdot y \cdot dy$

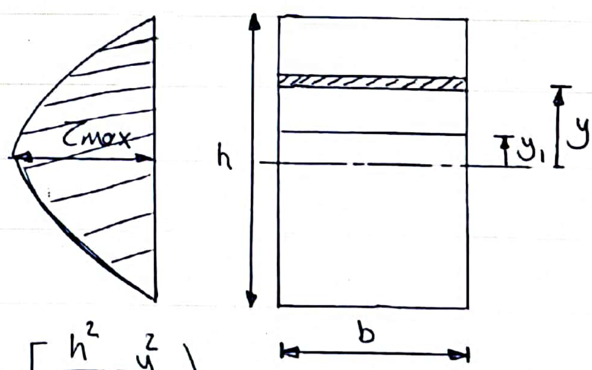
$$\Rightarrow \tau \cdot b_1 = \frac{Q}{I} \int_{y_1}^{h} b \cdot y \cdot dy = \frac{Q}{I} \int_{y_1}^{h} y \cdot dA$$

$$\Rightarrow \tau = \frac{Q A \bar{y}}{I b}$$

* Rectangular Section :-

$b = b_1$

$$\tau \cdot b = \frac{Q}{I} \int_{y_1}^{h/2} b \cdot y \cdot dy$$

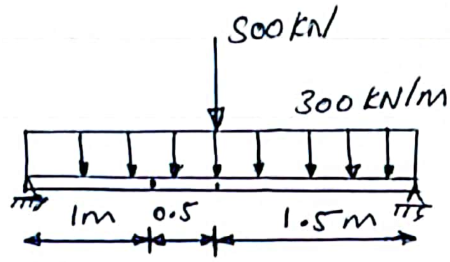


$$I = \frac{b h^3}{12}$$

$$\tau \cdot b = \frac{12\,Q}{b h^3} \left[\frac{y^2}{2} \right]_{y_1}^{h/2} = \frac{6Q}{b h^3} \left[\frac{h^2}{4} - y_1^2 \right)$$

at $y_1 = 0 \Rightarrow \tau = \tau_{max} = \frac{3}{2} \frac{Q}{b \cdot h} = \frac{3}{2} \tau_{mean}$.

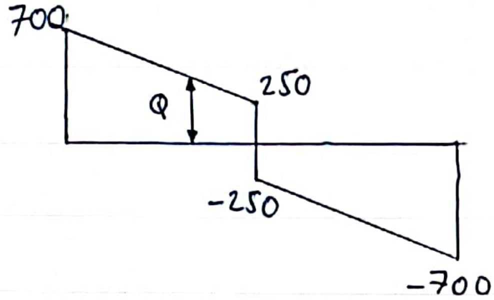
Example ① :- find shear stress at E and D for the cross-section as shown.



Sol:- $R_A = R_B = 700\ kN$

From S.F.D $\Rightarrow Q = 700 - 300*1$

$= 400\ kN$



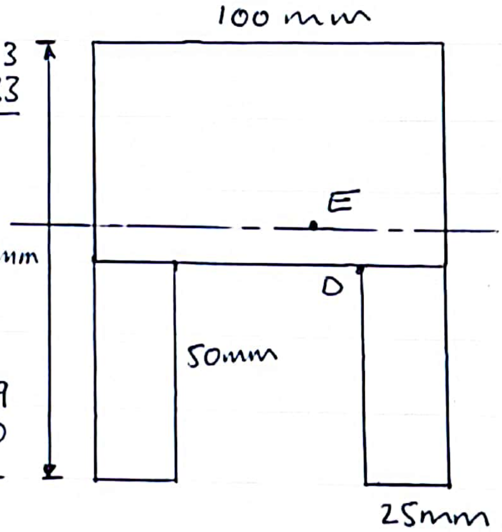
$$\bar{y} = \frac{\Sigma A y}{\Sigma A}$$

$$= \frac{100*100*50 - 50*50*25}{100*100 - 50*50}$$

$\Rightarrow \bar{y} = 58.33\ mm$

$$I = \frac{100*41.67^3}{3} + 2*\frac{25*58.33^3}{3} + \frac{50*8.33^3}{3}$$

$= 5.72 * 10^6\ mm^4$



$$\tau_E = \frac{Q A \bar{y}}{I \cdot b}$$

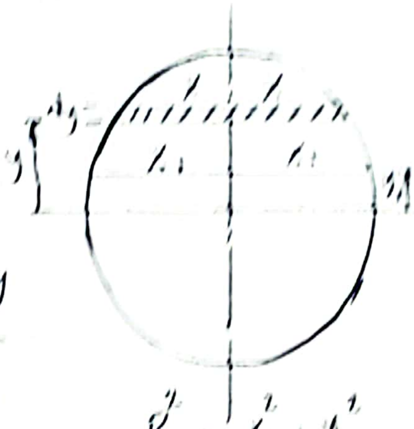
$$= \frac{400*10^3*(41.6*100)*20.83*10^{-9}}{5.72*10^{-6}*100*10^{-3}}$$

$= 60.6\ MPa.$

$$\tau_D = \frac{400*10^3*(50*100)*16.66*10^{-9}}{5.72*10^{-6}*100*10^{-3}} = 58.4\ Mpa.$$

$$\tau_D = \frac{400*10^3*50*100*16.66*10^{-9}}{5.72*10^{-6}*50*10^{-3}} = 116.8\ Mpa.$$

* Circular Section :-



$$\tau . b_1 = \frac{Q}{I} \int_{y_1}^{h} b . y . dy$$

$$\Rightarrow \tau . 2x_1 = \frac{Q}{I} \int_{y_1}^{R} 2x . y . dy$$

$$\Rightarrow \tau . 2\sqrt{R^2 - y_1^2} = \frac{4Q}{\pi R^4} \int_{y_1}^{R} 2\sqrt{R^2 - y^2} \, y . dy$$

$$\Rightarrow \tau . 2 \left(R^2 - y_1^2 \right)^{1/2} = - \frac{4Q}{\pi R^4} \left[(R^2 - y^2)^{3/2} . \frac{2}{3} \right]_{y_1}^{R}$$

$$\Rightarrow \tau . 2 (R^2 - y_1^2)^{1/2} = \frac{8Q}{3\pi R^4} (R^2 - y_1^2)^{3/2}$$

$$\Rightarrow \tau = \frac{4Q}{3\pi R^4} (R^2 - y_1^2)$$

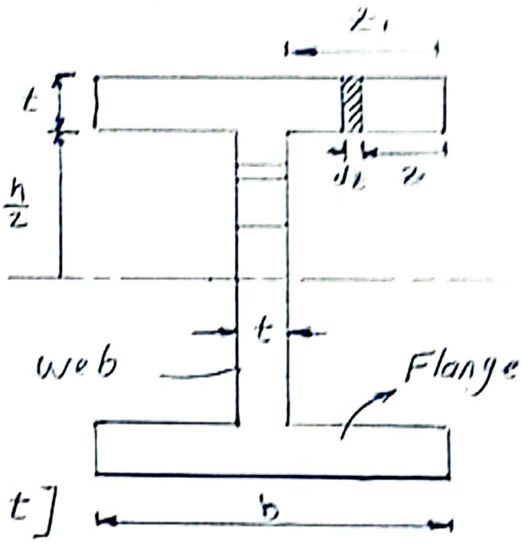
$R^2 = x_1^2 + y_1^2$

$R^2 = x^2 + y^2$

at $y_1 = 0 \Rightarrow \tau_{max} = \frac{4}{3} \frac{Q}{\pi R^2}$

$= \frac{4}{3} \tau_{mean}.$

* I - Section :- (web)



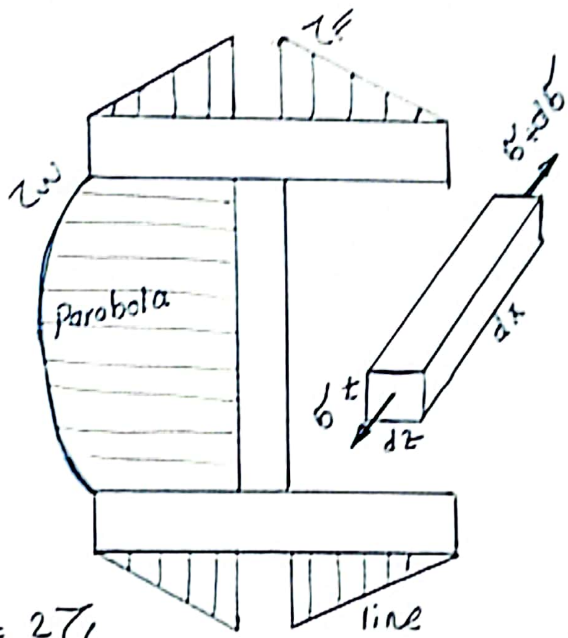
$$\tau . b_1 = \frac{Q}{I} \int_{y_1}^{h} y \, dA$$

$$\Rightarrow \tau . t = \frac{Q}{I} \left[\int_{y_1}^{h/2} y \, t . dy + \int_{h/2}^{h/2 + t} y \, b dy \right]$$

$$\Rightarrow \tau = \frac{Q}{2I} \left(\frac{h^2}{4} + bh + bt - y_1^2 \right]$$

at $y_1 = 0$

$$\Rightarrow \tau_{max} = \frac{Q}{2I} \left(\frac{h^2}{4} + bh + bt \right]$$



Flange :- $\tau . t . dx = \int_0^{z_1} d\sigma \, t . dz$

But $d\sigma = \frac{dM}{I} \left(\frac{h}{2} + \frac{t}{2} \right)$

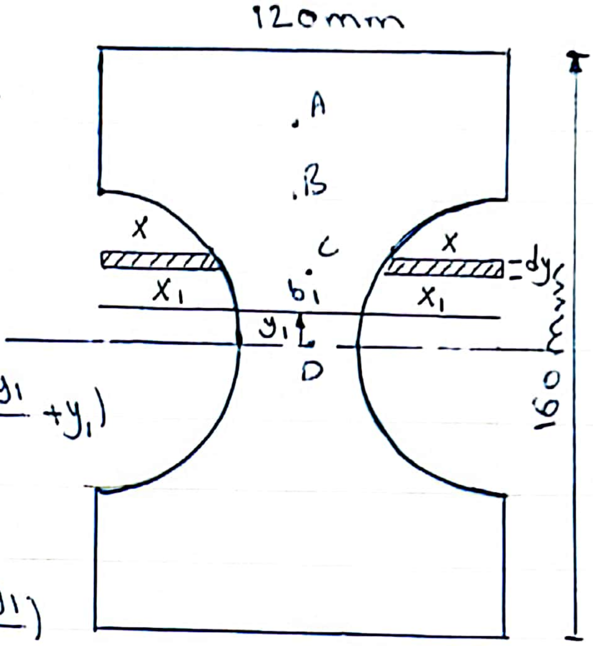
$\therefore \tau = \frac{Q}{2I} (h + t) \, z_1$

$$\tau_F = \frac{Q}{2I} (h + t) . \frac{b}{2}$$

$$\tau_w = \frac{Q}{2I} (h + t) \, b$$

Equilibrium of the junction common to the flange and the web: $\tau_w = 2\tau_f$

Example ② If $Q = 200$ kN find the shear stress at points A, B, C and D and also find the ratio between the maximum shear stress and mean shear stress.



Sol :- $I = \left[\frac{120 * 160^3}{12} - \frac{\pi}{64}(80)^4\right] * 10^{-12}$

$= 38.95 * 10^{-6}\ m^4$

$\tau \cdot (120 - 2x_1) = \frac{Q}{I}(A\bar{y}_{rec.} - A\bar{y}_{cir.})$

$\Rightarrow \tau\,(120 - 2x_1) = \frac{Q}{I}\left[120(80-y_1) * \left(\frac{80-y_1}{2} + y_1\right) - \int_{y_1}^{R} 2xy\,dy\right.$

$\Rightarrow \tau\,(120 - 2x_1) = \frac{Q}{I}\left[120(80-y_1)\left(\frac{80+y_1}{2}\right) - \int_{y_1}^{R} \sqrt{R^2-y^2}\,2y\,dy\right.$

$\Rightarrow 2\tau\left(60 - \sqrt{R^2-y_1^2}\right) = \frac{Q}{I}\left[60(80^2-y_1^2) - \left[-\frac{2}{3}(R^2-y^2)^{3/2}\right]_{y_1}^{R}\right.$

$= \frac{Q}{I}\left[60(80^2-y_1^2) - \left(-\frac{2}{3}\left[(R^2-R^2) - (R^2-y_1^2)\right]\right)\right.$

$= \frac{Q}{I}\left(60(80^2-y_1^2) - \left[\frac{2}{3}(R^2-y_1^2)^{3/2}\right]\right.$

$\Rightarrow \tau\left(60 - \sqrt{R^2-y_1^2}\right) = \frac{Q}{I}\left[(30(80^2-y_1^2) - \frac{1}{3}(R^2-y_1^2)^{3/2}\right]$

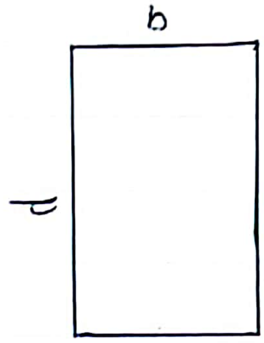
$\Rightarrow \tau\left(60 - \sqrt{40^2-y_1^2}\right) = \frac{200}{38.95 * 10^6}\left[30(80^2-y_1^2) - \frac{1}{3}(40^2-y_1^2)^{3/2}\right]$

at $y_1 = 0 \Rightarrow \tau_D = 43.816$ MPa , at $y_1 = 20$ mm $\Rightarrow \tau_C = 33.6$

at $y_1 = 40 \Rightarrow \tau_B = 12.3$ at $y_1 = 60 \Rightarrow \tau_A = 7.2$ MPa

$\tau_m = -$

Example (3) :- If a simply supported beam carry a concentrated load (W) at the center and the beam has a rectangular cross section with depth (d). At what distance be the shear stress equal to the mean shear stress?



Sol :- $\tau_m = \frac{Q}{A} = \frac{W}{2bd}$

$\tau = \frac{Q \cdot A \cdot \bar{y}}{I \cdot b}$. $A = b(\frac{d}{2} - y_1)$

$\bar{y} = \frac{d/2 - y_1}{2} + y_1 = \frac{d/2 + y_1}{2}$

$I = \frac{bd^3}{12}$

$\Rightarrow \tau = \frac{W \cdot b(\frac{d}{2} - y_1)(\frac{d}{2} + y_1) * 12}{2 * 2 \; bd^3 \cdot b} = \frac{3W(\frac{d^2}{4} - y_1^2)}{d^3 \cdot b}$

$\tau = \tau_{mean} \Rightarrow \frac{3W(\frac{d^2}{4} - y_1^2)}{d^3 \cdot b} = \frac{W}{2bd}$

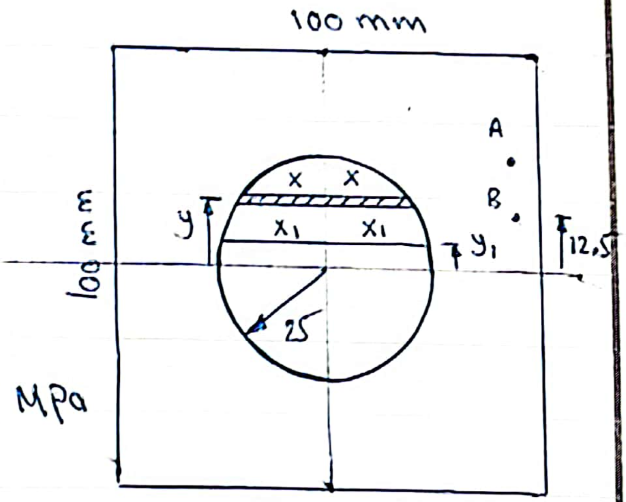
$\Rightarrow 6(\frac{d^2}{4} - y_1^2) = d^2 \Rightarrow 1.5d^2 - 6y_1^2 = d^2 \Rightarrow 6y_1^2 = \frac{d^2}{2} \Rightarrow y_1 = \frac{d}{\sqrt{12}}$

Example (4) :- $Q = 140$ kN.

Sol :-

$I = \left[\frac{100 * 100^3}{12} - \frac{\pi}{64}(50)^4\right]$

$= 8.02 * 10^6 \; mm^4$



$\tau_A = \frac{140 * (100 * 25) * 37.5}{8.02 \times 10^6 * 100} = 16.4$ MPa

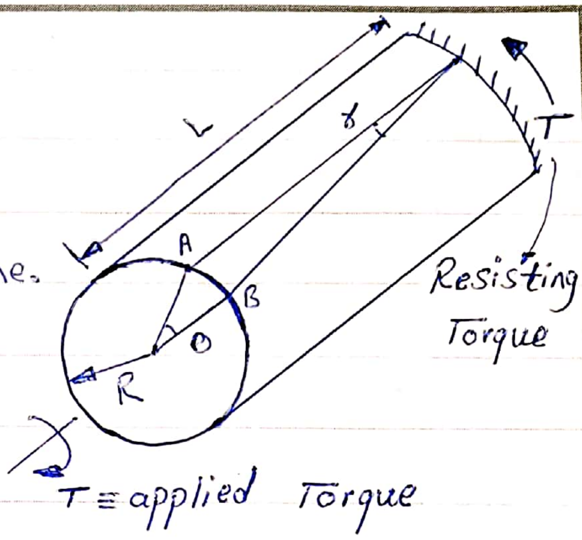
$\tau . b_1 = \frac{Q}{I} \int_{y_1}^{h} y \, dA \Rightarrow \tau(100 - 2X_1) = \frac{Q}{I}\left[100(50 - y_1)(50 - \frac{50 - y_1}{2}) - \int_{y_1}^{25} y(2x\,dy)\right]$

But :- $25^2 = X^2 + y^2 = X_1^2 + y_1^2$

* Simple torsion theory :-

* Assumptions :-

1. The material is homogeneous.
2. The load is within the elastic zone.
3. Circular sections remain circular.
4. Cross-section remain plane.
5. Cross-section rotate as if rigid.



* Derivation :- $\widehat{AB} = R\theta = \gamma L$ where :-

$R \equiv$ Radius of circular shaft.

$\theta \equiv$ Angle of twist (unit radian).

$\gamma \equiv$ Angle of distortion (shear strain).

$L \equiv$ Length of the twisted shaft.

But :- $\gamma = \dfrac{\tau}{G} \Rightarrow \dfrac{\tau}{G} \cdot L = R \cdot \theta \Rightarrow \dfrac{\tau}{R} = \dfrac{G \cdot \theta}{L} = \dfrac{\bar{\tau}}{r}$

Where :-

$\tau \equiv$ shear stress at radius (R).

$G \equiv$ Modulus of rigidity.

$\bar{\tau} \equiv$ shear stress at radius (r).

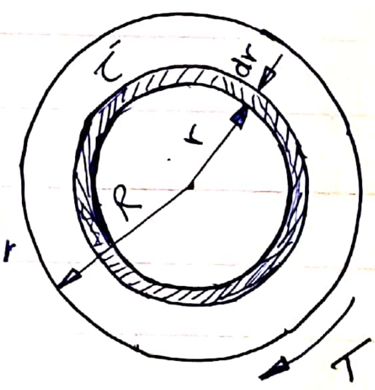
$dF = \bar{\tau} \times 2\pi r\, dr$

$dT = r \cdot dF = 2\pi r^2 \bar{\tau}\, dr$

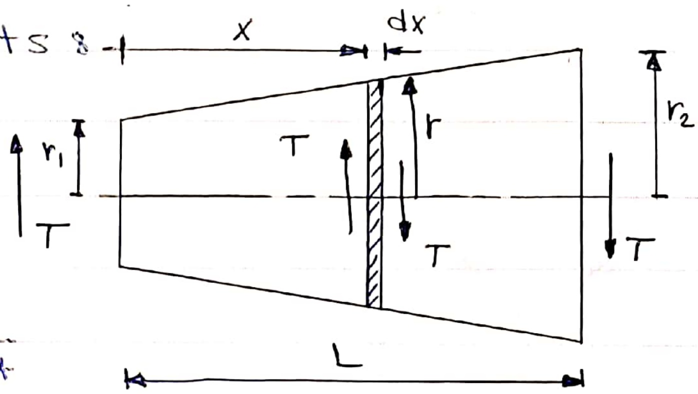
$\therefore \int dT = \int_0^R 2\pi r^2 \bar{\tau}\, dr = \int_0^R 2\pi \left(\dfrac{G \cdot \theta}{L} r\right) r^2\, dr$

$\Rightarrow T = \dfrac{G \cdot \theta}{L} \left(\dfrac{\pi}{2} R^4\right)$

$\therefore T = \dfrac{G \cdot \theta}{L} J \Rightarrow \dfrac{T}{J} = \dfrac{G \cdot \theta}{L}$ where



* Torsion of tapered shafts :-



$$r = r_1 + \frac{r_2 - r_1}{L} x$$

$$\frac{G \cdot \theta}{L} = \frac{T}{J} \Rightarrow \frac{G \cdot d\theta}{dx} = \frac{T}{\frac{\pi}{2} r^4}$$

$$\therefore \theta = \int d\theta = \int_0^L \frac{T}{G} \cdot \frac{dx}{\frac{\pi}{2}\left(r_1 + \frac{r_2 - r_1}{L} x\right)^4}$$

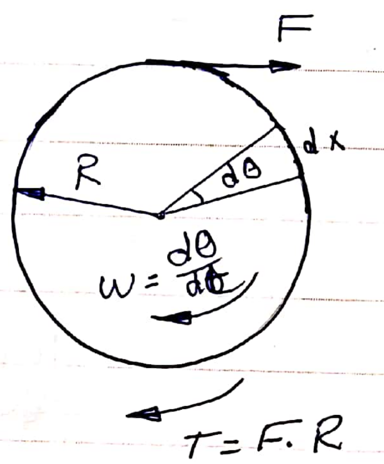
$$\Rightarrow \theta = \frac{T \cdot L}{G} \cdot \frac{2}{3\pi} \cdot \frac{r_1^2 + r_1 r_2 + r_2^2}{r_1^3 \cdot r_2^3}$$

if $r_1 = r_2 \Rightarrow \theta = \frac{T \cdot L}{G} \cdot \frac{1}{\frac{\pi}{2} R^4} = \frac{T \cdot L}{G \cdot J}$

* Power transmitted by shafts :-

$$\text{Power} = \frac{F \cdot dx}{dt} = \frac{T}{R} \cdot \frac{R\, d\theta}{dt}$$

$\therefore$ Power $= T \cdot \omega$ where :-



P ≡ Transmitted power (watt)

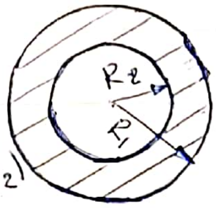
T ≡ Transmitted Torque (N.m)

ω ≡ Angular speed (rad/sec)

J = Polar Second moment of area.

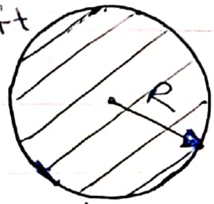
hollow shaft

$$J = \frac{\pi}{2}(R_1^4 - R_2^4)$$



Solid shaft

$$J = \frac{\pi}{2} R^4$$

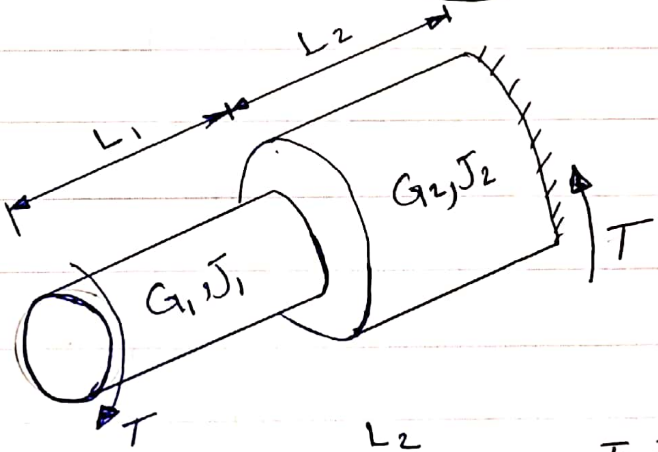


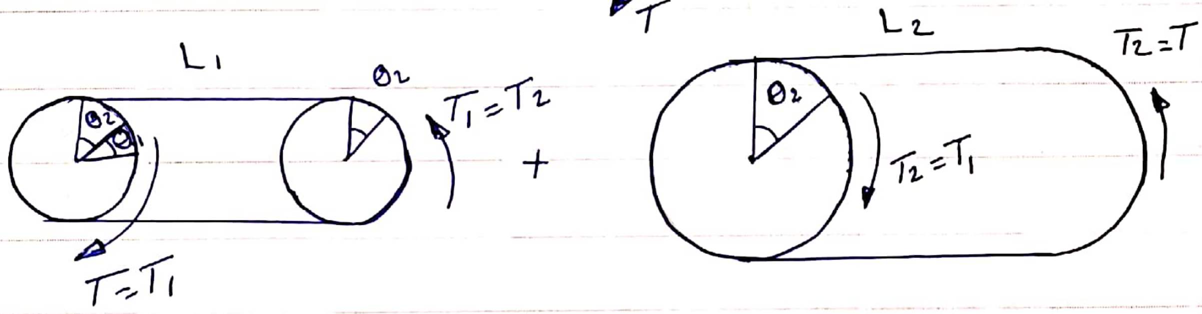
* Compound shafts :-

① Series Connection :-

$$\text{Total } \theta = \theta_1 + \theta_2$$

$$T = T_1 = T_2$$

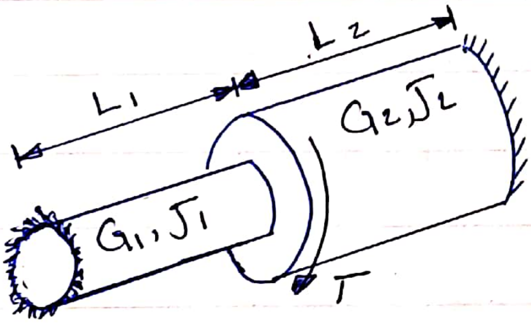


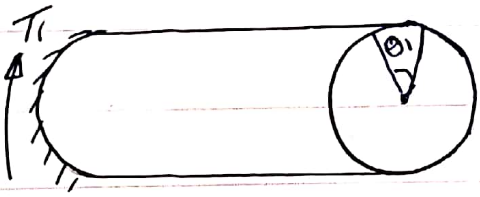


② Parallel Conection :-

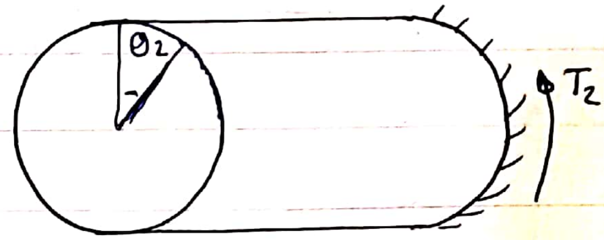
$$\text{Total } T = T_1 + T_2$$

$$\theta = \theta_1 = \theta_2$$





$$\theta_1 = \theta_2$$



Solution :-

$$\theta_1 = \theta_2 \Rightarrow \frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2} \Rightarrow \frac{1.8\, T_1}{\frac{\pi}{2}*25^4} = \frac{1.2\, T_2}{\frac{\pi}{2}*12.5^4}$$

$$\left.\begin{aligned} &\Rightarrow T_1 = 10.67\, T_2 \rightarrow (1) \\ &T_1 + T_2 = 680 \rightarrow (2) \end{aligned}\right] \Rightarrow T_1 = 622 \;,\; T_2 = 58 \text{ N.m}$$

$$\tau_1 = \frac{T_1 R_1}{J_1} = \frac{622 * 25 * 10^{-3}}{\frac{\pi}{2} * 25^4 * 10^{-12}} = 25.33 \text{ MPa}$$

$$\tau_2 = \frac{T_2 R_2}{J_2} = \frac{58 * 12.5 * 10^{-3}}{\frac{\pi}{2} * 12.5^4 * 10^{-12}} = 19 \text{ MPa}.$$

$$\theta = \theta_1 = \theta_2 = \frac{T_1 L_1}{J_1 G_1} = \frac{622 * 1.8}{\frac{\pi}{2} * 25^4 * 10^{-12} * 80 * 10^9} = 0.0228 \text{ rad} = 1.3^\circ$$

H.W :- A flanged coupling having six bolts placed at a pitch circle diameter of 180mm connects two lengths of solid steel shafting of the same diameter. The shaft is required to transmit 80 kw at 240 rev/min. Assuming the allowable intensities of shearing stress in the shaft and bolts are 75 MPa and 55 MPa respectively and the maximum torque is 1.4 times the mean torque, find :-

(a) the diameter of the shaft.

(b) the diameter of the bolts.

Example ① :- Determine the dimensions of a hollow shaft with a diameter ratio 3:4 which is to transmit 60 KW at 200 rev/min. The maximum shear stress in the shaft is limited to 70 MPa and the angle of twist to 3.8 in a length of 4m. For the shaft material G = 80 GPa.

Solution :- $P = T.\omega \Rightarrow T = P/\omega$ $\omega = \frac{2\pi}{60} * 200$

$$\Rightarrow T = \frac{60*10^3}{\frac{2\pi}{60}*200} = 2860 \text{ N.m}$$

$$J = \frac{T.R}{\tau} \Rightarrow \frac{\pi}{2}\left[R^4 - (0.75R)^4\right] = \frac{2860 . R}{70*10^6} \rightarrow R = 33.65 \text{ mm}$$

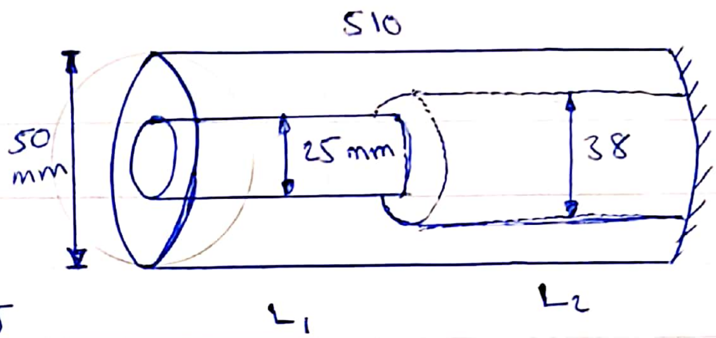
$$\theta = 3.8 * \frac{\pi}{180} \rightarrow J = \frac{T.L}{G.\theta} \Rightarrow \frac{\pi}{2}\left[R^4 - (0.75R)^4\right] = \frac{2860 * 4}{80*10^9 * 3.8 * \frac{\pi}{180}}$$

$$\Rightarrow R = 37.65 \text{ mm} \qquad \therefore R = 37.65 \text{ mm. (max)}$$

Example ② :- A shaft is (510 mm) long and (50 mm) external diameter. For part of its length it is bored to a diameter of 25 mm and for the rest to 38 mm diameter such that the angle of twist in both parts is the same. Find :-

① The maximum power transmitted at a speed of 210 rev/min if the shear stess is not exceed (700 MPa).

② The total angle of twist (G = 80 GPa).



Solution 8- ①

$$J = \frac{\pi}{2}(25^4 - 19^4) = 0.41 \times 10^6 \ mm^4$$

$$\frac{T \cdot R}{J} = \tau \Rightarrow T = \frac{\tau \cdot J}{R} = \frac{70 \times 10^6 \times 0.41}{25 \times 10^{-3}} = 1150 \ N.m$$

$$\text{Power} = T.\omega = 1150 \times \frac{2\pi \times 210}{60} = 25200 \ \text{watt}$$

② $\theta_1 = \theta_2 \Rightarrow \frac{T_1 L_1}{J_1 G_1} = \frac{T_2 L_2}{G_2 J_2} \Rightarrow \frac{L_1}{J_1} = \frac{L_2}{J_2}$

$$\Rightarrow \frac{L_1}{L_2} = \frac{J_1}{J_2} = \frac{\frac{\pi}{32}(50^4 - 25^4)}{\frac{\pi}{32}(50^4 - 38^4)} \Rightarrow L_1 = 1.43 L_2 \rightarrow (2)$$

$$L_1 + L_2 = 510 \rightarrow (2) \Rightarrow L_2 = 210 \quad L_1 = 300 \ mm$$

$$\theta = \theta_1 + \theta_2 = 2\theta_1$$

$$= \frac{2 T_1 L_1}{G_1 J_1} = \frac{2 \times 1150 \times 300 \times 10^{-3}}{80 \times 10^9 \times \frac{\pi}{32}(50^4 - 25^4)} = 0.93 \ rad \times 10^{-3}$$

=

Example ③ :- A circular bar (ABC), 3 m long is rigidly fixed at its ends (A) and (C). The portion (AB) is 1.8 m long and of 50 mm diameter, and (BC) is 1.2 m long and of 25 mm diameter. If a twisting moment of 680 N.m is applied at B, find the value of resisting moment at (A) and (C) and the maximum stress in each section of the shaft. what will be the angle of twist of each portion? $G = 80 \ GN/m^2$.